Reply to "Comments on Kullback-Leibler and renormalized entropies: Applications to electroencephalograms of epilepsy patients".

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Abstract

Kopitzki et al (preceeding comment) claim that the relationship between Renormalized and Kullback-Leibler entropies has already been given in their previous papers. Moreover, they argue that the first can give more useful information for e.g. localizing the seizure-generating area in epilepsy patients.

In our reply we stress that if the relationship between both entropies would have been known by them, they should have noticed that the condition on the effective temperature is unnecessary. Indeed, this condition led them to choose different reference segments for different channels, even if this was physiologically unplausible. Therefore, we still argue that it is very unlikely that renormalized entropy will give more information than the conventional Kullback-Leibler entropy.

We thank the authors of the preceding comment for pointing out a misprint in [1] (in the line following Eq. (8) it should read $p \equiv \tilde{q}$ instead of $p \equiv q$), and a numerical inconsistency in Figs. 2-4 of ref. [1]. The latter resulted from an error in our code. The corrected data is shown in Fig. 1 below, where we added the renormalized entropy values calculated with a pre-seizure reference for completeness. Note that Eq. (10) of [1], i.e. $|\Delta H| \leq K(p|q)$, is now verified in all cases [5]. Despite this correction, the data are qualitatively similar to the ones presented in [1], and we still conclude that renormalized entropy does not give more information than standard Kullback-Leibler (KL) entropy.

Apart from this, we do not agree with any of the other claims raised in the preceding comment (and we still have some discrepancy in details with the numerical results shown in [2] whose origin is unclear to us).

The first main point of ref. [1] was to show that the "renormalized entropy" (RE) proposed in [3] and applied to EEG data in [2] was indeed a KL entropy, but taking an unusual "renormalized" reference. We maintain, in contrast to claims made in the comment, that this relation (Eq. (9) in [1]) was not mentioned in [3, 4], and not in [2] either. Indeed, due to it, the condition $T_{eff} \geq 1$ postulated in [3, 4, 2] is not needed to obtain the inequality $\Delta H \leq 0$. The fact that the latter was claimed in [3, 4, 2] to hold only for $T_{eff} \geq 1$ indicates that the authors were not aware of the relation to KL (or "relative") entropy. Apart from this, we also wanted to give a simple treatment free of all allusions to statistical thermodynamics, the latter making the treatments in [3, 2] hard to understand.

Our second point was that RE is very unlikely to be more useful than the usual (un-renormalized) KL entropy for the analysis of EEGs from epileptic patients, as claimed in [2]. On the one hand this was based on the numerical similarity between RE and standard KL entropies, which is enforced by several inequalities and which makes it unlikely a priori that either is superior. On the other hand, we verified this explicitly by detailed numerical calculations which indeed showed that both behaved very similar. It is clear from Fig. 1 that major differences are due to the choice of the reference window. In contrast to what is suggested in the comment, we did

not base this conclusion entirely on theoretical arguments.

Finally, we also stressed that the condition $T_{eff} \geq 1$ – which is not needed at all – has led the authors of [2] to choose reference points which are physiologically very unfortunate. Again, we remark that it is very unreliable to compare a relative entropy measure obtained from EEG recordings at different electrodes by using different references (from the pre-seizure stage, during the seizure, or from the post-seizure stage) for each electrode in order to localize an epileptic focus. Thus the failure of realizing that RE is a sort of KL entropy – or at least of drawing the obvious consequences from this observation – has hampered its application to EEGs.

References

[1] R. Quian Quiroga, J. Arnhold, K. Lehnertz, and P. Grassberger, Phys. Rev. E **62**, 8382 (2000).

[2] K. Kopitzki, P.C. Warnke, and J. Timmer, Phys. Rev. E **58**, 4859 (1998).

[3] P. Saparin, A. Witt, J. Kurths, and V. Anishchenko, Chaos, Solitons and Fractals **4**, 1907 (1994).

[4] Yu. L. Klimontovich, Physica A **142**, 390 (1987).

[5] We do not understand how this relationship proved in ref. [1] can be invalidated by "basic mathematical operations" as claimed by the authors in their reference [9].
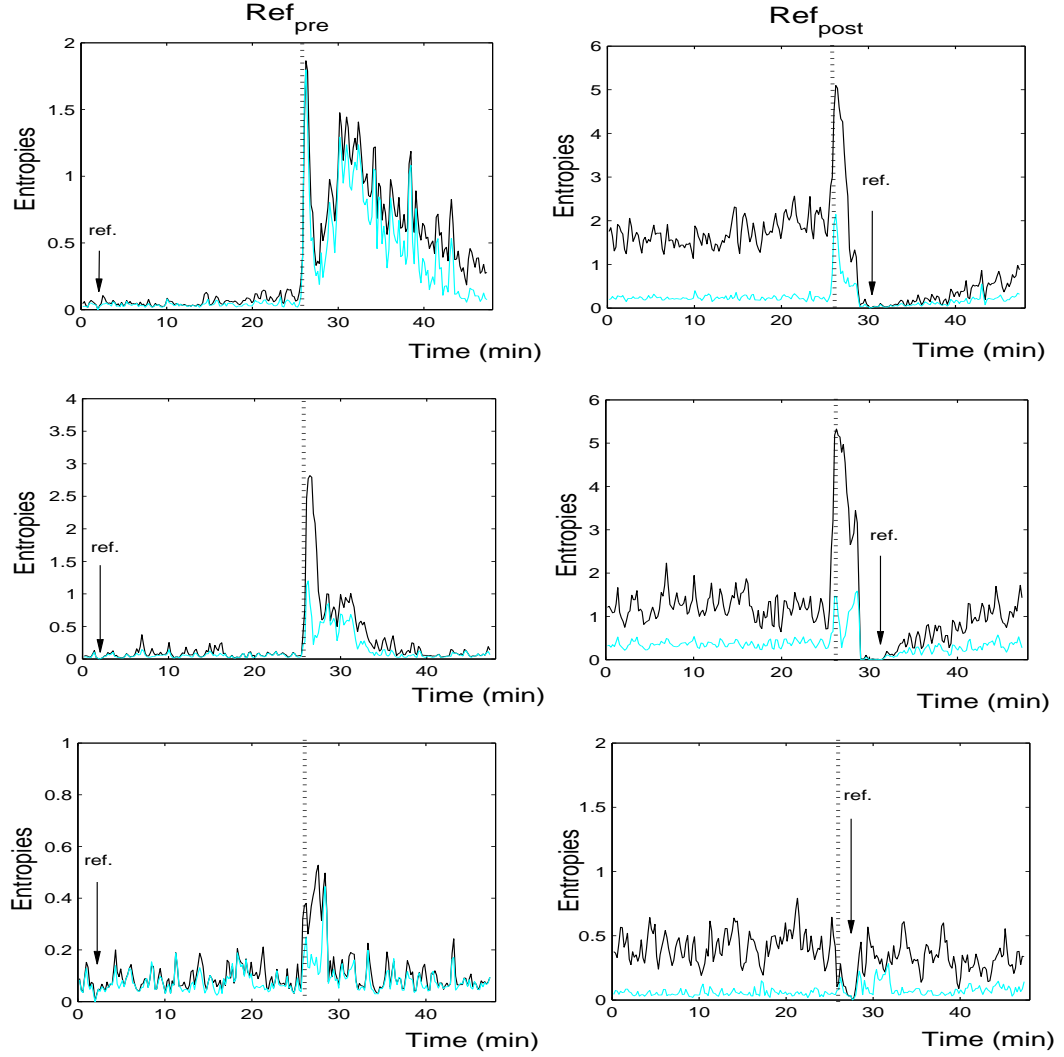
Figure 1: Kullback-Leibler (black line) and renormalized (gray line) entropies from EEGs recorded in the seizure-generating area (upper row), adjacent to the seizure generating areas (middle row), and in the non-affected brain hemisphere (lower row). Data shown in left (right) columns were obtained from using a pre- (post-) seizure reference window (marked by an arrow). The dotted vertical lines mark the electrical onset of the seizure.